

Masses and weak decay rates of doubly heavy baryons

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Abstract

Mass spectra and semileptonic decay rates of doubly heavy baryons are studied in the framework of the relativistic quark model in the quark-diquark approximation.

The description of doubly heavy baryon properties acquires in the last years the status of actual physical problem which can be studied experimentally. The appearance of experimental data on B_c mesons, heavy-light baryons stimulates the investigation of heavy quark bound systems and can help in discriminating numerous quark models. Recently first experimental indications of the existence of doubly charmed baryons were published by SELEX [1]. Although these data need further experimental confirmation and clarification it manifests that in the near future the masses and decay rates of doubly heavy baryons will be measured. This gives additional grounds for the theoretical investigation of the doubly heavy baryon properties. The success of the heavy quark effective theory (HQET) in predicting properties of the heavy-light $q\bar{Q}$ mesons (B and D) suggests to apply these methods to heavy-light baryons, too. The semileptonic decays of heavy hadrons present also an important tool for determining the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Doubly heavy baryons occupy a special position among existing baryons because they can be studied in the quark-diquark approximation and the

two-particle bound state methods can be applied. The two heavy quarks compose in this case a bound diquark system in the antitriplet colour state which serves as a localized colour source. The light quark q is orbiting around this heavy source at a distance much larger ($\sim 1/m_q$) than the source size ($\sim 2/m_Q$), see Fig. 1. The estimates of the light quark velocity in these baryons show that its value is $v/c \sim 0.7 - 0.8$ and the light quark should be treated fully relativistically. Thus the doubly heavy baryons look effectively like a two-body bound system and strongly resemble the heavy-light B and D mesons. Then the HQET expansion in the inverse heavy diquark mass can be performed. The ground state baryons with two heavy quarks can be composed from a compact doubly heavy diquark of spin 0 or 1 and a light quark. According to the Pauli principle the diquarks (bb) or (cc) have the spin 1 whereas diquark (bc) can have both the spin 0 and 1.

Here we study mass spectra and semi-leptonic decay rates of doubly heavy baryons using the relativistic quark model in the quark-diquark approximation.

In the quark-diquark picture of doubly heavy baryons the bound states of two heavy quarks and of the light quark and the heavy diquark are described by the diquark wave function (Ψ_d) and by the baryon wave function (Ψ_B), respectively. These wave functions satisfy the two-particle quasipotential equation of the Schrödinger type [2]

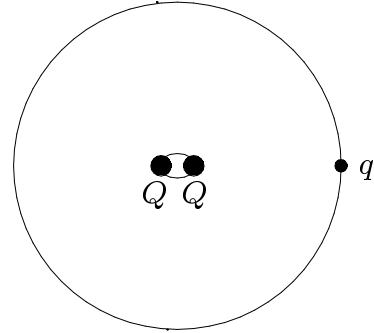


Figure 1: Schematic picture of doubly heavy baryon.

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and the center of mass energies of particles on the mass shell E_1 , E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \quad (3)$$

Here $M = E_1 + E_2$ is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of heavy quarks (Q_1 and Q_2) which form the diquark or of the heavy

diquark (d) and light quark (q) which form the doubly heavy baryon (B), and \mathbf{p} is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or quark-diquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Here we closely follow the similar construction of the quark-antiquark interaction in heavy mesons which were extensively studied in our relativistic quark model [3, 4]. For the quark-quark interaction in a diquark we use the relation $V_{QQ} = V_{Q\bar{Q}}/2$ arising under the assumption about the octet structure of the interaction. The quasipotential of the quark-antiquark interaction $V_{Q\bar{Q}}$ is the sum of the usual one-gluon exchange term and the confining part which is the mixture of long-range vector and scalar linear potentials, where the vector confining potential contains the Pauli term. The explicit expressions for V_{QQ} and V_{dq} and the details of the mass spectrum calculation are given in Ref. [5]. The quark masses have the following values $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV.

The masses of the ground state axial vector diquarks were found to be $M_{cc}^{AV} = 3.226$ GeV, $M_{bb}^{AV} = 9.778$ GeV, $M_{bc}^{AV} = 6.526$ GeV, and the mass of the scalar diquark $M_{bc}^S = 6.519$ GeV. We calculated the mass spectra of light-quark – heavy-diquark system first in the infinitely heavy diquark mass limit and then with the inclusion of $1/M_d$ corrections. In the infinitely heavy diquark mass limit its mass and spin decouple and the dynamics of the heavy hadron is determined by the light quark alone in accord with the heavy quark symmetry. Thus the properties of heavy-light mesons and doubly heavy baryons are similar in the limit $m_Q \rightarrow \infty$. Inclusion of the first order $1/m_Q$ corrections breaks the heavy quark symmetry and leads to different splittings in mesons and baryons [3, 5]. Note that in our calculations we treated the light quark completely relativistically without applying unjustified expansion in inverse powers of its mass.

The mass level orderings of Ξ_{cc} and Ξ_{bb} baryons are schematically shown in Fig. 2. There we first show our predictions for spectra in the limit when all $1/M_d$ corrections are neglected. In this limit the P -wave excitations of the light quark are inverted. This means that the mass of the state with higher light quark angular momentum $j = 3/2$ is smaller than the mass of

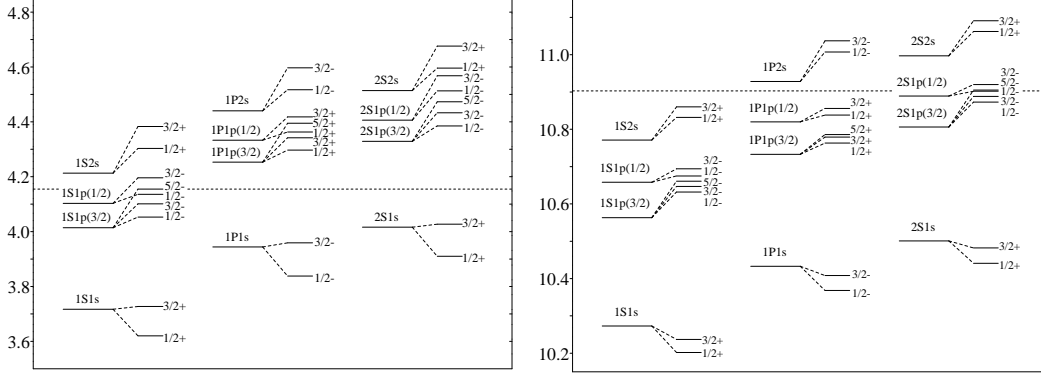


Figure 2: Masses of Ξ_{cc} (left) and Ξ_{bb} baryons (right) (in GeV).

the state with lower angular momentum $j = 1/2$. Next we switch on $1/M_d$ corrections. This results in splitting of the degenerate states and mixing of states with different j , which have the same total angular momentum J and parity. The fine splitting of P -levels turns out to be of the same order of magnitude as the gap between $j = 1/2$ and $j = 3/2$ degenerate multiplets in the $M_d \rightarrow \infty$ limit. The inclusion of $1/M_d$ corrections leads also to relative shifts of the hadron levels further decreasing this gap. As a result, some of the P -levels from different (initially degenerate) multiplets overlap; however, the centers of levels averaged over the heavy diquark spin remain inverted.

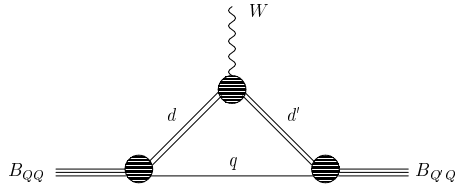


Figure 3: Weak transition matrix element of the doubly heavy baryon in the quark-diquark approximation.

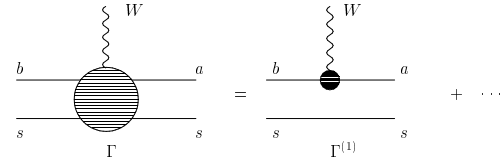


Figure 4: The leading order contribution $\Gamma^{(1)}$ to the diquark vertex function Γ .

The consideration of semileptonic decays of doubly heavy baryons (bbq) or (bcq) to doubly heavy baryons (bcq) or (ccq) can be divided into two steps (see Fig. 3). The first step is the study of form factors of the weak transition between initial and final doubly heavy diquarks. The second one consists in the inclusion of the light quark in order to compose a baryon with spin $1/2$ or $3/2$.

In the relativistic quark model the transition matrix element between two

diquark states is determined [6] by the convolution of the wave functions Ψ_d of the initial and final diquarks with the two particle vertex function Γ

$$\langle d'(Q)|J_\mu^W|d(P)\rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{d',Q}(\mathbf{q}) \Gamma_\mu(\mathbf{p}, \mathbf{q}) \Psi_{d,P}(\mathbf{p}). \quad (5)$$

Here $P = vM_i$, M_i , v denote the four-momentum, mass and four-velocity of the initial diquark $Q_b Q_s$ and $Q = v'M_f$, M_f , v' denote the four-momentum, mass and four-velocity of the final diquark $Q_a Q_s$; \mathbf{p} and \mathbf{q} are the relative quark-quark momenta.

The leading contribution to the vertex function Γ_μ comes from the diagram in Fig. 4 (index b denotes the initial active quark, index a the final active quark and index s the spectator)

$$\Gamma_\mu(\mathbf{p}, \mathbf{q}) = \Gamma_\mu^{(1)} = \bar{u}_a(\mathbf{q}_1) \gamma_\mu (1 - \gamma_5) u_b(\mathbf{p}_1) \bar{u}_s(\mathbf{q}_2) u_s(\mathbf{p}_2) (2\pi)^3 \delta(\mathbf{p}_2 - \mathbf{q}_2), \quad (6)$$

where $u(p)$ is the Dirac spinor.

The transformation of the bound state wave function from the rest frame to the moving one with four-momentum P is given by [6]

$$\Psi_{d,P}(\mathbf{p}) = D_b^{1/2}(R_{L_P}^W) D_s^{1/2}(R_{L_P}^W) \Psi_{d,0}(\mathbf{p}), \quad (7)$$

where R^W is the Wigner rotation, L_P is the Lorentz boost from the diquark rest frame to a moving one, and $D^{1/2}(R)$ is the rotation matrix.

Using this relation and the properties of the Dirac spinors and rotation matrices we can express the matrix element (5) in the form of the trace over spinor indices of both particles [7]. The final covariant expression for the transition matrix element reads

$$\frac{\langle d'(Q)|J_\mu^W|d(P)\rangle}{2\sqrt{M_i M_f}} = \int \frac{d^3p d^3q}{(2\pi)^3} \text{Tr} \{ \bar{\Psi}_{d'}(Q, q) \gamma_\mu (1 - \gamma_5) \Psi_d(P, p) \} \delta^3(\mathbf{p}_2 - \mathbf{q}_2), \quad (8)$$

where the amplitudes for the scalar (S) and axial vector (AV) diquarks (d) are given by

$$\begin{aligned} \Psi_S(P, p) = & \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \sqrt{\frac{\epsilon_s(p) + m_s}{2\epsilon_s(p)}} \left[\frac{\hat{v} + 1}{2\sqrt{2}} + \frac{\hat{v} - 1}{2\sqrt{2}} \frac{\tilde{p}^2}{(\epsilon_b(p) + m_b)(\epsilon_s(p) + m_s)} \right. \\ & \left. - \left(\frac{\hat{v} + 1}{2\sqrt{2}} \frac{1}{\epsilon_s(p) + m_s} + \frac{\hat{v} - 1}{2\sqrt{2}} \frac{1}{\epsilon_b(p) + m_b} \right) \hat{\tilde{p}} \right] \gamma_0 \Phi_S(p), \end{aligned} \quad (9)$$

$$\begin{aligned} \Psi_{AV}(P, p, \varepsilon) = & \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \sqrt{\frac{\epsilon_s(p) + m_s}{2\epsilon_s(p)}} \left[\frac{\hat{v} + 1}{2\sqrt{2}} \hat{\varepsilon} + \frac{\hat{v} - 1}{2\sqrt{2}} \right. \\ & \times \frac{\tilde{p}^2}{(\epsilon_b(p) + m_b)(\epsilon_s(p) + m_s)} \hat{\varepsilon} - \frac{\hat{v} - 1}{2\sqrt{2}} \frac{2(\varepsilon \cdot \tilde{p})\tilde{p}}{(\epsilon_b(p) + m_b)(\epsilon_s(p) + m_s)} \\ & \left. + \frac{\hat{v} + 1}{2\sqrt{2}} \frac{\hat{\varepsilon}\tilde{p}}{\epsilon_s(p) + m_s} - \frac{\hat{v} - 1}{2\sqrt{2}} \frac{\tilde{p}\hat{\varepsilon}}{\epsilon_b(p) + m_b} \right] \gamma_0 \gamma^5 \Phi_{AV}(p). \end{aligned} \quad (10)$$

Here $\Phi_d(p) \equiv \Psi_{d,0}(\mathbf{p})/\sqrt{2M_d}$ is the diquark wave function in the rest frame normalized to unity and the four-vector $\tilde{p} = L_P(0, \mathbf{p})$. The argument of the δ -function in Eq. (8) can be rewritten as

$$\mathbf{p}_2 - \mathbf{q}_2 = \mathbf{q} - \mathbf{p} - \frac{\epsilon_s(p) + \epsilon_s(q)}{w + 1}(\mathbf{v}' - \mathbf{v}), \quad (11)$$

where $w = (v \cdot v')$. The spectator quark contribution factors out in all decay matrix elements. They have a common factor

$$\begin{aligned} & \sqrt{\frac{\epsilon_s(p) + m_s}{2\epsilon_s(p)}} \sqrt{\frac{\epsilon_s(q) + m_s}{2\epsilon_s(q)}} \left[1 - \sqrt{\frac{w - 1}{w + 1}} \left(\frac{\sqrt{\mathbf{p}^2}}{\epsilon_s(p) + m_s} + \frac{\sqrt{\mathbf{q}^2}}{\epsilon_s(q) + m_s} \right) \right. \\ & \quad \left. + \frac{\sqrt{\mathbf{p}^2}\sqrt{\mathbf{q}^2}}{[\epsilon_s(q) + m_s][\epsilon_s(p) + m_s]} \right] = \sqrt{\frac{2}{w + 1}} I_s(p, q). \end{aligned} \quad (12)$$

If the δ -function is used to express \mathbf{q} through \mathbf{p} or \mathbf{p} through \mathbf{q} then $I_s(p, q) = \mathcal{I}_s(p)$ or $I_s(p, q) = \mathcal{I}_s(q)$ with

$$\begin{aligned} \mathcal{I}_s(p) = & \sqrt{\frac{w\epsilon_s(p) - \sqrt{w^2 - 1}\sqrt{\mathbf{p}^2}}{\epsilon_s(p)}} \theta \left(\sqrt{\epsilon_s(p) - m_s} - \sqrt{\frac{w - 1}{w + 1}} \sqrt{\epsilon_s(p) + m_s} \right) \\ & + \frac{m_s}{\sqrt{\epsilon_s(p)[w\epsilon_s(p) - \sqrt{w^2 - 1}\sqrt{\mathbf{p}^2}]}} \theta \left(\sqrt{\frac{w - 1}{w + 1}} \sqrt{\epsilon_s(p) + m_s} - \sqrt{\epsilon_s(p) - m_s} \right). \end{aligned}$$

The weak current matrix elements, e.g., for the scalar to axial vector diquark transition ($bc \rightarrow cc$) have the following covariant decomposition

$$\frac{\langle AV(v', \varepsilon') | J_\mu^V | S(v) \rangle}{\sqrt{M_{AV} M_S}} = i h_V(w) \epsilon_{\mu\alpha\beta\gamma} \varepsilon'^{\alpha} v'^{\beta} v^{\gamma}, \quad (13)$$

$$\frac{\langle AV(v', \varepsilon') | J_\mu^A | S(v) \rangle}{\sqrt{M_{AV} M_S}} = h_{A_1}(w)(w + 1)\varepsilon'_\mu - h_{A_2}(w)(v \cdot \varepsilon'^*)v_\mu - h_{A_3}(v \cdot \varepsilon'^*)v'_\mu. \quad (14)$$

These transition form factors are expressed through the overlap integrals of the diquark wave functions and are given in Ref. [7]. These exact expressions for diquark form factors were obtained without any assumptions about the spectator and active quark masses. For the heavy diquark system we can apply the v/c expansion. Then in the nonrelativistic limit we get the following expressions for the form factors

$$\begin{aligned} h_V(w) &= [1 + (w + 1)f(w)]F(w), \\ h_{A_1}(w) &= h_{A_3}(w) = [1 + (w - 1)f(w)]F(w), \\ h_{A_2}(w) &= -2f(w)F(w), \end{aligned} \quad (15)$$

where

$$\begin{aligned} F(w) &= \sqrt{\frac{1}{w(w+1)}} \left(1 + \frac{m_a}{\sqrt{m_a^2 + (w^2 - 1)m_s^2}} \right)^{1/2} \\ &\times \int \frac{d^3p}{(2\pi)^3} \bar{\Phi}_F \left(\mathbf{p} + \frac{2m_s}{w+1}(\mathbf{v}' - \mathbf{v}) \right) \Phi_I(\mathbf{p}) \end{aligned} \quad (16)$$

and

$$f(w) = \frac{m_s}{\sqrt{m_a^2 + (w^2 - 1)m_s^2} + m_a}. \quad (17)$$

The appearance of the terms proportional to the function $f(w)$ is the result of the account of the spectator quark recoil. Their contribution is important and distinguishes our approach from the previous considerations [8, 9]. We plot the function $F(w)$ for $bb \rightarrow bc$ and $bc \rightarrow cc$ diquark transitions in Fig. 5. The function $F(w)$ falls off rather rapidly, especially for the $bb \rightarrow bc$ diquark transition where the spectator quark is the b quark. Such a decrease is the consequence of the large mass of the spectator quark and the high recoil momentum ($q_{\max} \approx m_b - m_c \sim 3.33$ GeV) transferred.

The second step in studying weak transitions of doubly heavy baryons is the inclusion of the spectator light quark in the consideration. We carry out all further calculations in the limit of an infinitely heavy diquark, $M_d \rightarrow \infty$, treating the light quark relativistically. The transition matrix element between doubly heavy baryon states in the quark-diquark approximation (see Figs. 3 and 4) is given by [cf. Eqs. (5) and (6)]

$$\frac{\langle B'(Q) | J_\mu^W | B(P) \rangle}{2\sqrt{M_I M_F}} = \int \frac{d^3p d^3q}{(2\pi)^3} \bar{\Psi}_{B',Q}(\mathbf{q}) \langle d'(Q) | J_\mu^W | d(P) \rangle \Psi_{B,P}(\mathbf{p}) \delta^3(\mathbf{p}_q - \mathbf{q}_q), \quad (18)$$

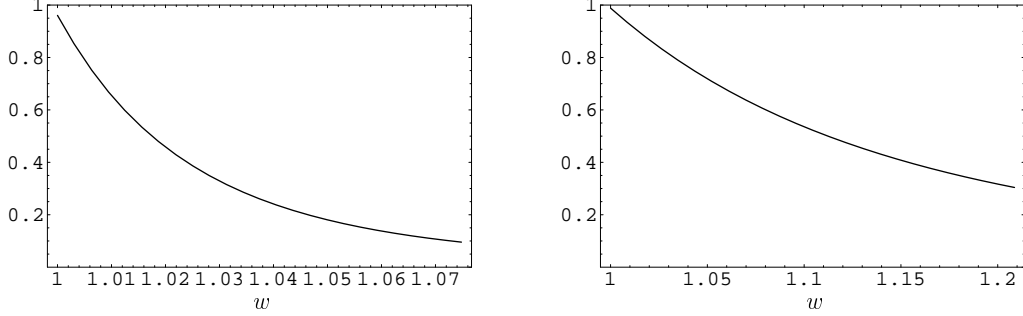


Figure 5: The function $F(w)$ for the $bb \rightarrow bc$ (left) and $bc \rightarrow cc$ (right) quark transitions.

where $\Psi_{B,P}(p)$ is the doubly heavy baryon wave function; \mathbf{p} and \mathbf{q} are the relative quark-diquark momenta. The baryon ground-state wave function $\Psi_{B,P}(p)$ is a product of the spin-independent part $\Psi_B(p)$ satisfying the related quasipotential equation (1) and the spin part $U_B(v)$

$$\Psi_{B,P}(\mathbf{p}) = \Psi_B(\mathbf{p})U_B(v). \quad (19)$$

The baryon spin wave function U_B is constructed from the Dirac spinor $u_q(v)$ of the light spectator quark and the diquark spin wave function. The ground state spin 1/2 baryons can contain either the scalar or axial vector diquark. The former baryon is denoted by $\Xi'_{QQ'}$ and the latter one by $\Xi_{QQ'}$. The ground state spin 3/2 baryon can be formed only from the axial vector diquark and is denoted by $\Xi_{QQ'}^*$.

The amplitude for the $\Xi'_{QQ_s} \rightarrow \Xi_{Q'Q_s}$ transition in the infinitely heavy diquark limit is given by the following expression

$$\frac{\langle \Xi_{Q'Q_s}(v') | J_\mu^W | \Xi'_{QQ_s}(v) \rangle}{2\sqrt{M_I M_F}} = \frac{i}{\sqrt{3}} [ih_V(w)\epsilon_{\mu\alpha\beta\gamma}v'^\beta v^\gamma - g_{\mu\alpha}h_{A_1}(w+1) + v_\mu v_\alpha h_{A_2}(w) + v'_\mu v_\alpha h_{A_3}(w)] \bar{U}_{\Xi_{Q'Q_s}}(v')\gamma_5(\gamma^\alpha + v'^\alpha)U_{\Xi'_{QQ_s}}(v)\eta(w), \quad (20)$$

where $\eta(w)$ is the heavy diquark – light quark Isgur-Wise function which is determined by the dynamics of the light spectator quark q

$$\eta(w) = \sqrt{\frac{2}{w+1}} \int \frac{d^3p d^3q}{(2\pi)^3} \bar{\Psi}_B(\mathbf{q})\Psi_B(\mathbf{p})I_q(p,q) \times \delta^3\left(\mathbf{p} - \mathbf{q} + \frac{\epsilon_q(p) + \epsilon_q(q)}{w+1}(\mathbf{v}' - \mathbf{v})\right). \quad (21)$$

We plot the Isgur-Wise function $\eta(w)$ in Fig. 6. In the nonrelativistic limit for heavy quarks the diquark form factors $h_i(w)$ contain the common factor $F(w)\eta(w)$.

Our results for the semileptonic decay rates of doubly heavy baryons Ξ_{bb} and Ξ_{bc} are compared with previous predictions in Table 1. The results of different approaches differ substantially. Most of previous papers [10, 9, 11] give their predictions only for selected decay rates.

Their values agree with our in the order of magnitude. Our predictions are smaller than the QCD sum rule results [11] by a factor of ~ 2 . This can be a result of our treatment of the heavy spectator quark recoil in the heavy diquark. On the other hand the authors of Ref. [12] using for calculations the Bethe-Salpeter equation give more decay channels. Their results are substantially higher than ours, for some decays the difference reaches almost two orders of magnitude which seems quite strange. E.g., for the sum of the semileptonic decays $\Xi_{bb} \rightarrow \Xi_{bc}^{(\prime,*)}$ Ref. [12] predicts $\sim 6 \times 10^{-13}$ GeV which almost saturates the estimate of the total decay rate $\Gamma_{\Xi_{bb}}^{\text{total}} \sim (8.3 \pm 0.3) \times 10^{-13}$ GeV [13] and thus is unlikely.

This work was supported in part by the DFG grant No. Eb 139/2-2.

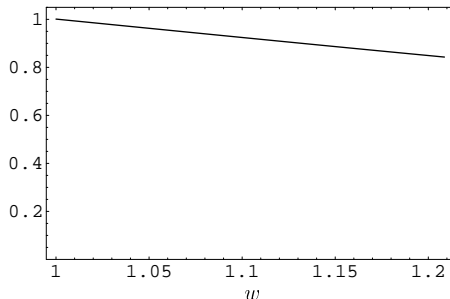


Figure 6: The Isgur-Wise function $\eta(w)$ of the light quark – heavy diquark bound system.

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Table 1: Semileptonic decay rates of doubly heavy baryons Ξ_{bb} and Ξ_{bc} (in $\times 10^{-14}$ GeV).

Decay	our	Ref.[12]	Ref.[9]	Ref.[11]	Ref.[10]
$\Xi_{bb} \rightarrow \Xi'_{bc}$	1.64	4.28			
$\Xi_{bb} \rightarrow \Xi_{bc}$	3.26	28.5		8.99	
$\Xi_{bb} \rightarrow \Xi_{bc}^*$	1.05	27.2		2.70	
$\Xi_{bb}^* \rightarrow \Xi'_{bc}$	1.63	8.57			
$\Xi_{bb}^* \rightarrow \Xi_{bc}$	0.55	52.0			
$\Xi_{bb}^* \rightarrow \Xi_{bc}^*$	3.83	12.9			
$\Xi'_{bc} \rightarrow \Xi_{cc}$	1.76	7.76			
$\Xi'_{bc} \rightarrow \Xi_{cc}^*$	3.40	28.8			
$\Xi_{bc} \rightarrow \Xi_{cc}$	4.59	8.93	4.0	8.87	0.8
$\Xi_{bc} \rightarrow \Xi_{cc}^*$	1.43	14.1	1.2	2.66	
$\Xi_{bc}^* \rightarrow \Xi_{cc}$	0.75	27.5			
$\Xi_{bc}^* \rightarrow \Xi_{cc}^*$	5.37	17.2			

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